ON GRADED QUATERNION-SYMBOL EQUIVALENCE

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Witt ring WK of a given field K gathers information about classes of admissible orthogonal geometries over K. Thus, two fields with isomorphic Witt rings (such fields are then said to be *Witt equivalent*) admit the same classes of orthogonal geometries notwithstanding possible differences in their underlying arithmetics. The pursue for efficient criteria for Witt equivalence dates back to early 1970s.

One very successful approach (appearing under different names: *reciprocity equivalence*, *Hilbert-symbol equivalence*, *quaternion-symbol equivalence*) relates Witt equivalence to a set of "local" isomorphisms between subgroup of Brauer groups of completions of two fields K and L with respect to certain subsets of their valuations. In principle this boils down to controlling splitting of 2-fold Pfister forms over the completions.

In this talk we will propose an alternative condition (called *graded quaternion-symbol equivalence*, hereafter), where the focus is shifted from Brauer groups to Brauer–Wall groups. The new condition is in general weaker, since with the graded quaternion-symbol equivalence we effectively control only binary form and not the 2-fold Pfister forms as before. It is not difficult to give an example of two fields for which exists a *graded* quaternion-symbol equivalence but not quaternion-symbol equivalence. Nevertheless, we will show that for some classes of fields these two notions coincide. The classes we are going to discuss are:

- global function fields,
- global number fields with unique dyadic completions,
- formally real algebraic function fields in one variable over real closed field.

This leads to a new criterion for Witt equivalence of fields belonging to these classes.