PERFECTOID FIELDS, DEEPLY RAMIFIED FIELDS AND THEIR RELATIVES

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Perfectoid fields have come to fame as they constitute the basis for the perfectoid spaces introduced and studied by Scholze. They allow a tight connection between certain fields of characteristic 0 with valuations of positive residue characteristic and corresponding valued fields of positive characteristic via the tilting construction. However, they are not really suitable for our research, as they are complete of rank 1 and hence not first order axiomatizable. A much better class is that of *deeply ramified fields* (in the sense of the book "Almost ring theory" of Gabber and Ramero). From their definition we have derived two other classes of valued fields: the *semitame fields* and the *generalized deeply ramified fields*, and we have studied their valuation theoretical properties.

The reason for our research are the following deep open problems in positive characteristic:

1) local uniformization, the local form of resolution of singularities, in arbitrary dimension,

2) decidability of the field $\mathbb{F}_q((t))$ of Laurent series over a finite field $\mathbb{F}_q$, and of its perfect hull.

Both problems are connected with the structure theory of valued function fields of positive characteristic. The main obstruction here is the phenomenon of the *defect* of finite extensions of valued fields.

Defects have been classified; Anna Blaszczok will report on this in her talk. The independent defects, which constitute one of the two classes, have been found to be the more "harmless" ones. We have proved that deeply ramified fields allow only independent defects, and we used this property to characterize all of the above mentioned classes of valued fields.

The semitame fields constitute an interesting generalization of the *tame fields*, which have been used to prove partial results related to the above stated problems. It is now our hope that these results can be generalized to semitame fields by showing that the independent defects are indeed harmless and can be overcome through further research of

their structure.

This is joint work with Anna Blaszczok