PRÜFER DOMAINS OF INTEGER-VALUED POLYNOMIALS OVER SUBSETS

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Let V be a rank one valuation domain with quotient field K. Recently, Loper and Werner characterized the pseudo-convergent sequences $E = \{s_n\}_{n\in\mathbb{N}} \subset V$ in the sense of Ostrowski for which the ring

$$\mathrm{Int}(E, V) = \{f \in K[X] \mid f(E) \subseteq V\}$$

of integer-valued polynomials over E is a Prüfer domain. In particular, their result shows that there are subsets E of V which are not precompact but $\mathrm{Int}(E, V)$ is Prüfer (e.g.: take E to be a pseudo-convergent sequence of transcendental type and non-zero breadth ideal). In this talk we generalize Loper and Werner's result to a general subset S of V and give a complete characterization of when $\mathrm{Int}(S, V)$ is Prüfer. We show first that the obstruction for such a ring to be Prüfer is the presence of valuation overrings which are residually transcendental extensions of V, and then we show that this happens when S contains a pseudo-monotone sequence in the sense of Chabert, which generalizes pseudo-convergent sequences. Finally, we prove that $\mathrm{Int}(S, V)$ is Prüfer if and only if no element of the algebraic closure of K is a pseudo-limit of a pseudo-monotone sequence of elements of S, with respect to some extension of V.