

EXTENSIONS OF VALUATIONS DOMINATING A NOETHERIAN DOMAIN TO THE HENSELIZATION AND COMPLETION

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Suppose that (R, m_R) is a local domain which is dominated by a valuation ν . The associated graded ring of R along ν , defined by Teissier, is

$$\mathrm{gr}_\nu(R) = \bigoplus_{\gamma \in \Phi_\nu} \mathcal{P}_\gamma(R)/\mathcal{P}_\gamma^+(R) = \bigoplus_{\gamma \in S^R(\nu)} \mathcal{P}_\gamma(R)/\mathcal{P}_\gamma^+(R)$$

where

$$\mathcal{P}_\gamma(R) = \{f \in R \mid \nu(f) \geq \gamma\} \text{ and } \mathcal{P}_\gamma^+(R) = \{f \in R \mid \nu(f) > \gamma\}.$$

We show that a natural generalization of local uniformization is false in general for valuations of rank larger than one, even in equicharacteristic zero. Suppose that R is an excellent local domain which is dominated by a valuation ν . We show that there does not always exist a regular local ring R' which birationally dominates R and is dominated by ν and an extension of ν to the Henselization $(R')^h$ of R' such that the associated graded rings of R' and $(R')^h$ along the valuations are equal. We also show that there does not always exist a regular local ring R' which birationally dominates R , a prime ideal p of the completion \widehat{R}' such that $p \cap R' = (0)$ and an extension of ν to \widehat{R}' such that the associated graded rings of R' and \widehat{R}'/p along the valuation are equal.