# A BOHR-JESSEN TYPE THEOREM FOR THE EPSTEIN ZETA-FUNCTION

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### Abstract

Let $Q$ be a positive definite quadratic $n \times n$ matrix. For $\mathbf{x} \in \mathbb{Z}^n$, define $Q[\mathbf{x}] = \mathbf{x}^T Q\mathbf{x}$. The Epstein zeta-function $\zeta(s; Q)$, $s = \sigma + it$, associated to $Q$ is defined, for $\sigma > \frac{n}{2}$, by

$$\zeta(s; Q) = \sum_{\mathbf{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} (Q[\mathbf{x}])^{-s},$$

and can be analytically continued to the whole complex plane, except for a simple pole at the point $s = \frac{n}{2}$ with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$. This function was introduced by P. Epstein in 1903.

In the second decade of the last century, H. Bohr proposed to apply probabilistic methods for the investigations of the value distribution of the Riemann zeta-function. His idea was realized in joint papers with B. Jessen [1], [2].

Our report is devoted to a Bohr-Jessen type theorem for the function $\zeta(s; Q)$. We suppose additionally that $Q[\mathbf{x}] \in \mathbb{Z}$ for every $\mathbf{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}$, and that $n \geq 4$ is even. Then the following statement is true.

**Theorem.** *Suppose that $\sigma > \frac{n-1}{2}$ is fixed. Then*

$$\frac{1}{T} meas \{t \in [0, T] : \zeta(\sigma + it; Q) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

*converges weakly to an explicitly given probability measure $P_\sigma$ as $T \to \infty$.*

Here $measA$ denotes the Lebesgue measure of a measurable set $A \subset \mathbb{C}$, $\mathcal{B}(\mathbb{C})$ is the Borel $\sigma$-field of the complex space, and $P_\sigma$ is the distribution of a certain complex-valued random element.

**R**eferences

1. H. Bohr, B. Jessen, Über die Wertverteilung der Riemanschen Zetafunktion, Erste Mitteilung, *Acta Math.* **54** (1930), 1–35.

2. H. Bohr, B. Jessen, Über die Wertverteilung der Riemanschen Zetafunktion, Zweite Mitteilung, *Acta Math.* **58** (1932), 1–55.

**This is joint work with Antanas Laurinčikas**