KEY POLYNOMIALS IN SIMPLE EXTENSIONS OF VALUED FIELDS AND SIMULTANEOUS EMBEDDED LOCAL UNIFORMIZATION IN CHARACTERISITC ZERO (FOLLOWING JULIE DECAUP).

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Let $\iota : K \hookrightarrow L = K(x)$ be a simple extension of valued fields. Let us denote by ν the given valuation on L, as well as its restriction to K. The notion of **key polynomials** for the extension ι was introduced by Saunders MacLane in the nineteen thirties in the special case when $\nu|_K$ is a discrete rank one valuation. The case of simple extensions of valued fields with arbitrary (K, ν) was studied much later by Michel Vaquié and, independently, by F.J. Herrera Govantes, W. Mahboub, M.A. Olalla Acosta and M. Spivakovsky. Recently, together with J. Decaup, W. Mahboub and J. Novacoski we have greatly simplified our version of the theory of key polynomials, at least the part of it needed for the proof of the Local Uniformization Theorem. In this lecture we will discuss the simplified presentation of the theory.

The problem of resolution of singularities is the problem of constructing, for every algebraic variety X (more generally, for any quasi-excellent noetherian scheme), a non-singular algebraic variety (resp. scheme) X' and a birational proper morphism $X' \to X$ that induces an isomorphism above the non-singular locus of X. The local version of the problem is often stated in terms of valuations. Let (R, M, k) be a local quasi-excellent noetherian domain. Let R_ν be a valuation ring birationally dominating R. The problem is to construct a regular local ring R' of finite type over R such that $R \subset R' \subset R_\nu$. The statement asserting the existence of such an R' is known as the **L**ocal Uniformization Theorem; it was proved by Zariski when *char* $k = 0$ and is an important open problem when *char* $k = p > 0$. A stronger version of this result, also known in characteristic zero, is **embedded local uniformization** or **monomialization**: given a regular local noetherian ring R, an element $f \in R$ and a valuation ν, centered at R, there exists a sequence $R \to R'$ of local blowings up along non-singular centers with respect to ν such that f is a monomial with respect to a suitable regular system of parameters of R'.

In this talk we will expalin the new definition of key polynomials and discuss their main properties. We will then proceed to an application to Local Uniformization and explain the main theorem of J. Decaup's Ph.D. thesis.

Theorem. *Let k be a ground field of characteristic zero and R a regular local finitely generated k-algebra. Let ν be a valuation centered at R such that the residue field of the valuation ring $k_\nu = k$. There exists an infinite sequence*

$$R \to R_1 \to \dots \to R_i \to \dots \tag{1}$$

of local blowings up along non-singular centers with respect to ν such that for every $f \in R$ there exists $i \in \mathbb{N}$ such that f is a monomial with respect to a suitable regular system of parameters of R_i.

The important (and surprising) point in this theorem is the fact that the sequence (1) does not depend on the chosen element $f \in R$ and works *simultaneously* for all the elements of R (hence the name "simultaneous embedded local uniformization").

This is joint work with J. Decaup, W. Mahboub, J. Novacoski