NATURAL HOMOMORPHISM OF WITT RINGS AND EQUIVALENT CONDITIONS FOR A DEDEKIND DOMAIN

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Let P and R be commutative rings with identity elements. Every homomorphism $P \to R$ between P and R induces a homomorphism $WP \to WR$ between their Witt rings. The homomorphism $WP \to WR$ is said to be *natural*, if it is induced by an embedding $P \hookrightarrow R$. In the case when P is a Dedekind domain and $R = K$ is its field of fractions, the natural homomorphism $WP \to WK$ is injective.

The injectivity of the natural homomorphism $WP \to WK$ is a necessary condition for P to be a Dedekind domain. But we know that it is not a sufficient condition. In the talk we give an example of a non-maximal order P in a quadratic number field K for which $WP \to WK$ is injective.

All this raises a natural question: what do we need to add to the condition of the injectivity of the natural homomorphism to get equivalent conditions for a Dedekind domain? We answer this question in the case of orders in number fields.